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## TARGET MATHEMATICS <br> THE EXCELLENCE KEY <br> AGYAT GUPTA (M.Sc., M.Phil.) <br> CODE:1301-AG-TS-2 REGNO:TMC-D/79/98/366/63

## General Instructions:-

(i) All Question are compulsory :
(ii) This question paper contains 29 questions.
(iii) Question 1-4 in Section $A$ are very sort-answer type question carrying 1 mark each
(iv) Question 5-12in Section B are sort-answer type question carrying $\mathbf{2}$ mark each.
(v) Question 13-23 in Section C are long-answer-I type question carrying 4 mark each
(vi) Question 24-29 in Section D are long-answer-II type question carrying 6 mark each
(vii) There is no overall choice. However, internal choice has been provided in 3 question of four marks and 3 questions of six marks each. You have to attempt only one lf the alternatives in all such questions.
(viii) Use of calculator is not permitted.
(ix) Please check that this question paper contains 6 printed pages.
(x) Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

## PRE-BOARD EXAMINATION 2018-19

CLASS - XII CBSE MATHEMATICS
PART - A (Question 1 to 4 carry 1 mark each.)
Q. $1 \quad$ If $A=\left(a_{i j}\right)$ is a matrix of order $2 \times 2$, such that $|A|=-15$ and $C_{i j}$ represents the cofactor of $a_{i j}$ then find $a_{21} c_{21}+a_{22} c_{22}$
Q. 2

For the curve $y=5 x-2 x^{3}$ if $x$ increases at the rate of 2 units/sec, then how fast is the slope of curve changing when $x=3$ ?

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| Q. 3 | Find the integrating factor for the linear differential equation $\left(y^{2}-1\right)+2 x y \frac{d y}{d x}=\left(\frac{2}{y^{2}-1}\right) \frac{d y}{d x}$. |
| :---: | :---: |
| Q. 4 | Find the acute angle which the line with direction cosines $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}, \mathrm{n}$ makes with positive direction of z -axis. <br> OR <br> Find the acute angle between the plane $5 x-4 y+7 z-13=0$ and the $y$-axis. |
|  | PART - B (Question 5 to 12 carry 2 mark each.) |
| Q. 5 | Let $A=\{1,2,3,4\}$. Let $R$ be the equivalence relation on $A \times A$ defined by $(a, b) R(c, d)$ iff $a+d=b+c$. Find the equivalence class $[(1,3)]$. <br> OR <br> Determine whether the binary operation * on the set $\mathbf{N}$ of natural numbers defined by $a * b=2^{a b}$ is associative or not. |
| Q. 6 | Find the matrix $X$ for which $\left[\begin{array}{cc}2 & -1 \\ 3 & 4\end{array}\right] X=\left[\begin{array}{cc}3 & 0 \\ 43 & 22\end{array}\right]$. |
| Q. 7 | Evaluate: $\int_{-\pi / 2}^{\pi / 2} \frac{x^{2}}{1+5^{x}} d x$. |
| Q. 8 | Evaluate: $\int \frac{\left(x^{2}+\cos ^{2} x\right) \operatorname{cosec}^{2} x}{1+x^{2}} d x$. <br> OR <br> Evaluate: $\int \frac{\sqrt{x^{2}+1}\left[\log \left(x^{2}+1\right)-2 \log x\right]}{x^{4}} d x$ |
| Q. 9 | Form the differential equation of the family of circle in the second quadrant and touching the coordinate axes. |

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Q. 10 If $\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $|2 \hat{a}+\hat{b}+\hat{c}|$.

## OR

Find $\lambda$ when the projection of $\hat{i}+\lambda \hat{j}+\hat{k}$ on $\hat{i}+\hat{j}$ is $\sqrt{2}$ units.
Q. 11 A pair of fair dice is thrown. Find the probability that the sum is 10 or greater, if 5 appears on the first die.
Q. 12 Two cards are drawn without replacement from a well shuffled pack of 52 cards. Find the probability that one is a king and other is a queen of opposite color.

## PART - C (Question 13 to 23 carry 4 mark each.)

Q. 13 If $a>b>c>0$, prove that $\cot ^{-1}\left(\frac{1+a b}{a-b}\right)+\cot ^{-1}\left(\frac{1+b c}{b-c}\right)+\cot ^{-1}\left(\frac{1+c a}{c-a}\right)=\pi$.
Q. 14 Prove that : $\left|\begin{array}{lll}-2 a & a+b & a+c \\ b+a & -2 b & b+c \\ c+a & c+b & -2 c\end{array}\right|=4(b+c)(c+a)(a+b)$.
Q. 15 If $y=x \log \left(\frac{x}{a+b x}\right)$ then, prove that $x^{3} \frac{d^{2} y}{d x^{2}}=\left(x \frac{d y}{d x}-y\right)^{2}$.

## OR

If $x=a \sin 2 t(1+\cos 2 t)$ and $y=b \cos 2 t(1-\cos 2 t)$, then find $\frac{d y}{d x} a t=\frac{\pi}{4}$.
Q. 16 Determine the values of ' $a$ ' and ' $b$ ' such that the following function is

$$
\left\{\begin{array}{c}
\frac{x+\sin x}{\sin (a+1) x}, \text { if }-\pi<x<0 \\
2, \text { if } x=0 \\
2 \frac{e^{\sin b x}-1}{b x}, \text { if } x>0
\end{array}\right.
$$

Q. 17 Find the point on the curve $9 y^{2}=x^{3}$, where the normal to the curve makes equal intercepts on the axes.

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| Q. 18 | Evaluate : $\int_{0}^{\pi / 4} \frac{\sec \mathrm{x}}{1+2 \sin ^{2} \mathrm{x}} \mathrm{dx}$ |
| :--- | :--- |
| Q. 19 | Evaluate : $\int \frac{x^{4}+1}{x\left(1+x^{2}\right)^{2}} d x$ |

Q. 20 Show that the equation of a plane, which meets the axes in A, B and C and the given centroid of the triangle ABC is the point $(\alpha, \beta, \gamma)$, is $\frac{\mathrm{x}}{\alpha}+\frac{\mathrm{y}}{\beta}+\frac{\mathrm{z}}{\gamma}=3$.
If $3 p$ is distance of plane from origin, show that $\alpha^{-2}+\beta^{-2}+\gamma^{-2}=p^{-2}$.
Q. 21

Find the general solution of the differential equation: $\frac{d x}{d y}=\frac{y \tan y-x \tan y-x y}{y \tan y}$ OR
Solve the differential equation $\left[\frac{e^{-2 \sqrt{y}}}{\sqrt{y}}-\frac{x}{\sqrt{y}}\right] \frac{d y}{d x}=1 ;(\mathrm{y} \neq 0)$ and $y(1)=2$

Vectors $\vec{a}, \vec{b}, \vec{c}$ are of the same magnitude and taken pairwise in order form equal angles. If $\overrightarrow{\mathrm{a}}=\hat{\mathrm{i}}+\hat{\mathrm{j}}$ and $\overrightarrow{\mathrm{b}}=\hat{\mathrm{j}}+\hat{\mathrm{k}}$ find $\overrightarrow{\mathrm{c}}$.
Q. 23

Show that function $f: R \rightarrow\{x \in R:-1<x<1\}$ defined by
$f(x)=\frac{x}{1+|x|}, x \in R$, is one-one $\&$ onto function.
OR
Let $\mathrm{A}=\mathrm{W} \times \mathrm{W}$ and let * be a binary operation on A defined by $(\mathrm{a}, \mathrm{b}) *(\mathrm{c}, \mathrm{d})=$ $(\mathrm{ad}+\mathrm{bc}, \mathrm{bd})$ for all $(\mathrm{a}, \mathrm{b}),(\mathrm{c}, \mathrm{d})(\mathrm{a}, \mathrm{b}),(\mathrm{c}, \mathrm{d}) \in \mathrm{W} \times \mathrm{W}$.
(1) Show that * is commutative on A.
(2) Show that * is associative on A.
(3) Find the identity element of * in A.

PART - D (Question 24 to 29 carry 6 mark each.)

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Q. 24

If $A=\left[\begin{array}{ll}0 & 1 \\ 0 & 0\end{array}\right]$, prove that $(a I+b A)^{n}=a^{n} . I+n a^{n-1} b A$ where $I$ is a unit matrix of order 2 and n is a positive integer.

OR
Two trusts A \& B receive Rs. 70000 and 55000 respectively from central government to award prize to persons of a district in 3 fields agriculture, education and social services. Trust a awarded 10, 5 and 15 persons in the field of agriculture, education and social services respectively while trust B awarded 15, 10 and 5 persons in the field of agriculture, education and social services respectively. If all three prizes together amount to Rs. 6000, them find amount of each prize by matrix method.
Q. 25 A cylinder of greatest volume is inscribed in a cone, show that Volume of the cylinder $=\frac{4}{27} \pi h^{3} \tan { }^{2} \alpha$. Where $\mathrm{r}, \mathrm{h}, \alpha$ are the radius, height and semi - vertical angle of the cone and $R, H$ are the radius and height of the inscribed cylinder.
Q. 26 Using integration, find the area of the triangle bounded by the lines $x+2 y=$ $2, \mathrm{y}-\mathrm{x}=1$ and $2 \mathrm{x}+\mathrm{y}=7$.

## OR

Using integration, find the area of the region $\left\{(x, y): x^{2}+y^{2} \leq 1 \leq x+\frac{y}{2} ; x, y \in R\right\}$
Q. 27 The members of a consulting firm rent cars from three rental agencies: $50 \%$ from agency X, $30 \%$ from agency Y and $20 \%$ from agency Z . Form past experience it is known that $9 \%$ of the cars from agency X need a service and tuning before renting, $12 \%$ of the cars from agency Y need a service and tuning before renting and $10 \%$ of the cars from agency Z need a service and turning before renting. If the rental car delivered to the firm needs service and tuning, find the probability that agency Z is not to blamed.

## OR

A shopkeeper sells three types of flower seeds A1, A2 and A3. They are sold as a mixture where the proportions are 4:4:2 respectively. The

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germination rates of three types of seeds are $45 \%, 60 \%$ and $35 \%$. Calculate the probability (a) of a randomly chosen seed to germinate.(b) that it is of the type A2, given that a randomly chosen seed does not germinate.

Find the direction ratios of the normal to the plane, which passes through the points $(1,0,0)$ and $(0,1,0)$ and makes angle $\frac{\pi}{4}$ which the plane $\mathrm{x}+\mathrm{y}=3$. Also find the equation of the plane. OR
Show that the line of intersection of the planes $x+2 y+3 z=8$ and $2 x+3 y+4 z=11$ is coplanar with the line $\frac{x+1}{1}=\frac{y+1}{2}=\frac{z+1}{3}$. Also find the equation of the plane containing them.
Q. 29 An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirements are $4500 \mathrm{~L}, 3000 \mathrm{~L}$ and 3500 L respectively. The distance (in km ) between the depots and the petrol pumps is given in the following table:

| Distance in (km) |  |  |
| :---: | :---: | :---: |
| From/To | A | B |
| D | 7 | 3 |
| E | 6 | 4 |
| F | 3 | 2 |

Assuming that the transportation cost of 10 litres of oil is Re 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?
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यदि आप दृढ संकल्प और पूर्णता के साथ काम करेंगे तो सफलता ज़रूर मिलेगी

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